STATISTICS USING ANALOG TECHNIQUES

Calculation of Exponentially Mapped Past Statistical Variables Using a True RMS-to-DC Converter

by Dr. R. H. Smallwood*

It is often useful to be able to calculate both the average frequency of a signal and the way the average frequency varies with time, that is, the *variance* of the average. The calculation of the Exponentially Mapped Past (EMP) statistical variables [1] and their use with the electrical signals from the human stomach [2,3] have appeared in the literature.

The EMP average frequency is given by

$$\overline{f_\tau(0)} = \frac{1}{\tau}\int_{-\infty}^{0} f(t)\,\epsilon^{-t/\tau}dt \qquad (1)$$

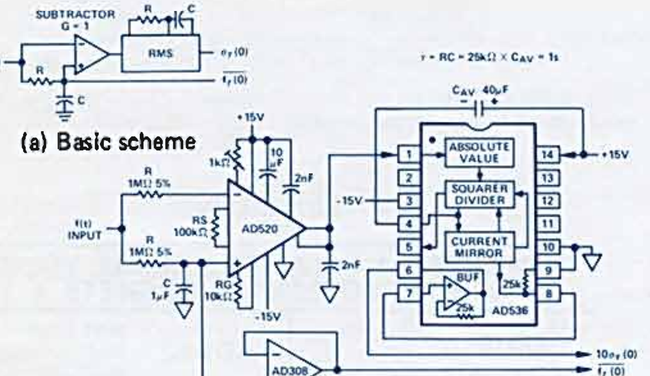
where τ is the weighting (RC) time constant. (1) will be recognized as the RC average of the input frequency f(t)†. The EMP variance is given by

$$\sigma_\tau^2(0) = \int_{-\infty}^{0}\left[f(t) - \overline{f_\tau(t)}\right]^2 \epsilon^{-t/\tau}dt \qquad (2)$$

where f(t) is the instantaneous frequency and $\overline{f_\tau(t)}$ is the EMP average frequency. As the variance is the mean-square value of a zero-mean process and

$$\int_{-\infty}^{0}\left[f(t) - \overline{f_\tau(t)}\right] = 0 \qquad (3)$$

it will be seen that the EMP standard deviation, $\sigma_\tau(0)$, is given by the rms value of the difference between the instantaneous and the mean frequency. This is shown symbolically in Figure 1a, and in circuit form using an AD520§ as the difference amplifier and the AD536§ as the rms-to-dc converter, in Figure 1b. The input is from a circuit that calculates the instantaneous frequency of the signal using an AD7520 DAC[4].



(b) Details of circuit employed

Figure 1. Scheme for measuring average and standard deviation.

The circuit of Figure 1b has been used to calculate the average frequency and its variance for the electrical signals, at about 0.05Hz, which are produced the by human stomach. In this example, they were recorded on tape at 15/16in/s and replayed at 60in/s, giving a frequency of about 3.2Hz (upper trace in Figure 2). A phase-locked-loop system tracked the signal, and the loop oscillator's instantaneous frequency was found by counting the number of cycles of a 1kHz oscillator between successive positive-going transitions of the signal. This gives a measure of each period of the input signal, and a voltage proportional to *frequency* was obtained using the AD7520 as a divider[4] (second trace, Figure 2).

The EMP average frequency is given by the buffered output of the RC filter on the lagged input of the AD520 (third trace in Figure 2). The amplified difference between the two inputs (10V F.S., gain of 10) is rms'ed in the AD536, with a time-constant of τ. The rms output is fourth trace in Figure 2.

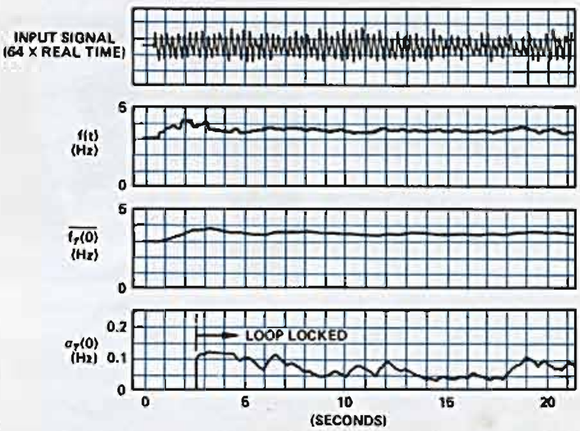


Figure 2. Chart recordings of variables (a) measured electrical activity, (b) frequency (input to EMP computer), (c) average frequency, (d) variance function (amplified). Real time scale — 64s/s.

The time constant was set at 1s (3.2Hz input) so that the EMP average was effectively taken over the last 10 cycles of the signal. Using a Student's t test, it could be demonstrated (less than 5% probability of pure chance) that the frequency of the electrical signals from the human stomach changes over a period of a few minutes. The biological meaning of this observation is obscure!

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†Here, the variable we are considering is not the voltage or current of a signal, but its *frequency* (see top waveforms in Figure 2)

§The AD521 and the AD536A, more-recent designs, would be used in future implementations of this technique. Use the reply card for technical data.

[1] J. Otterman, "The Properties and Methods for Computation of Exponentially Mapped Past Statistical Variables, *IRE Trans.* AC-5, 11-17, 1960

[2] T.S. Nelson & T.M. Wallace, "An Instrumentation System for On-Line Processing and Statistical Analysis of Electroenterographic and Electrogastrographic Data." *Digest of the 6th International Conference on Medical Electronics*, Tokyo, 1965, 369-370

[3] R.H. Smallwood, "Analysis of Gastric Electrical Signals from Surface Electrodes Using Phaselock Techniques, I. System Design, II. System Performance with Gastric Signals." *Med. & Biol. Eng., 1978*

[4] R.H. Smallwood, "An Instantaneous Ratemeter for Low-Frequency Signals", *Electronic Engineering 50*, 27, 1978